INVESTIGATION OF HEAT TRANSFER AND HYDRODYNAMIC DRAG IN THE TURBULENT FLOW OF A GAS IN THE FIELD OF A LONGITUDINAL PRESSURE GRADIENT OF VARIABLE SIGN. I

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We present results from a study of heat transfer and hydrodynamic drag in the turbulent flow of a gas through a channel formed by a series of flat nonsymmetric diffusers with a divergence angle of 12° and a series of converging diffuser sections, for the interval Re = $(10-80) \cdot 10^3$.

The problem of intensifying the transfer of heat that takes place between a gas and a surface is presently solved primarily through the use of various types of extended surfaces, which in addition to developing the surface result in an intensification of the transfer processes as a consequence of periodic mechanical separation of the boundary layer. The use of turbulization mechanisms for the solution of this problem, as demonstrated experimentally [1], is effective in a region of relatively low values, i.e., Re = $(3-20) \cdot 10^3$. Intensification of heat transfer can be achieved by setting up transverse pressure gradients in the flow, and these bring about the continuous displacement of the boundary layer by the external flow [2].

This paper is devoted to an investigation of the transfer of heat and the hydrodynamic drag in the turbulent flow of a gas through a channel, in the presence of longitudinal pressure gradients of variable sign.

The gas flow through a diffuser (i.e., in the field of a positive pressure gradient) is accompanied by a substantial increase in the coefficient of turbulent momentum transfer [3, 4]. Here, in accordance with the concept of identity for momentum and heat carriers, we should expect a noticeable intensification of heat transfer.

Nevertheless, we know [5] that the gas flow through the converging section of a diffuser (i.e., in the field of a longitudinal negative pressure gradient) is associated with a reduction in heat-transfer intensity, and this is explained by the cessation of turbulence generation and the degeneration of the residual turbulence as a result of the negative pressure gradients – an effect that is well known from [6].

We are naturally led to the idea of studying a channel which is made up of a series of diffusers with small divergence angles, and a series of converging diffuser sections. The turbulent energy in such a channel, accumulated by the flow in the diffuser, is used profitably in the converging section. It goes without saying that the extent of the diffuser portions of the channel, even in the case of small divergence angles, must be limited both to reduce the volume of the heat-exchange apparatus and to prevent flow separation – an effect which results in great energy losses.

We have studied the case of a channel which is made up of a series of flat nonsymmetric diffusers with γ = 12° and a series of converging diffuser sections; this channel is formed by two copper plates (a flat and a shaped plate) with h = 300 mm and l = 960 mm (Fig. 1).

The plates which form the diverging and converging sections of the diffusers (to make up the channel) represent the structural part of hollow chambers which were placed into an open wind tunnel operating in suction. The air flow initially passed through the hydrodynamic stabilization segment, subsequently entering

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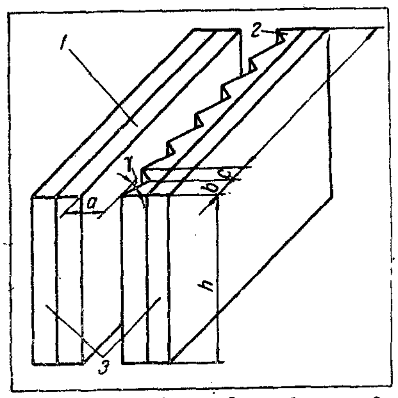

Fig. 1. A channel made up of diverging and converging diffuser sections: 1) flat plate; 2) shaped plate; 3) hollow chamber.

the channel being investigated. The thermal-stabilization segment was not turned on prior to the test, since its function is insignificant under conditions involving flow with elevated turbulence, and given sufficiently high values for l/d_{eq}, its effect cannot possibly be felt.

The air was heated by condensation of the water vapor at atmospheric pressure within the hollow chambers on the inside of the plate whose surface, facing the air, was thus virtually isothermal. The air flow rate was measured by means of a precalibrated pneumometric tube installed at the inlet to the wind-tunnel collector shaped to conform to the arc of a circle.

The air temperature at the inlet to and at the outlet from the channel was measured with copper–constantan thermocouples; an air mixer was set up at the channel outlet. The thermocouple readings were checked by means of back-up laboratory mercury thermometers graduated to 0.1°C.

We determined the heat flow from the air and it was monitored by checking the quantity of condensate collected on the flat and shaped plates in separate measuring flasks. The time to fill the flasks was determined with a stopwatch giving readings of 0.2 sec.

The static pressure differences were measured with a miniscope accurate to 0.01 mm water column.

The vapor saturation temperature was taken as the wall temperature. Correspondingly, the average coefficient of heat transfer between the wall and the air, for the channel as a whole, was assumed to be equal to the experimentally determined heat-transfer coefficient.

We used experimental data to determine the following values:

$$\mathrm{Re} \equiv \frac{w d_{eq}}{\nu}, \quad \zeta = \frac{\Delta p}{\frac{1}{2}\rho w^2} \frac{d_{eq}}{l}, \quad \mathrm{Nu} \equiv \frac{\alpha d_{eq}}{\lambda}.$$

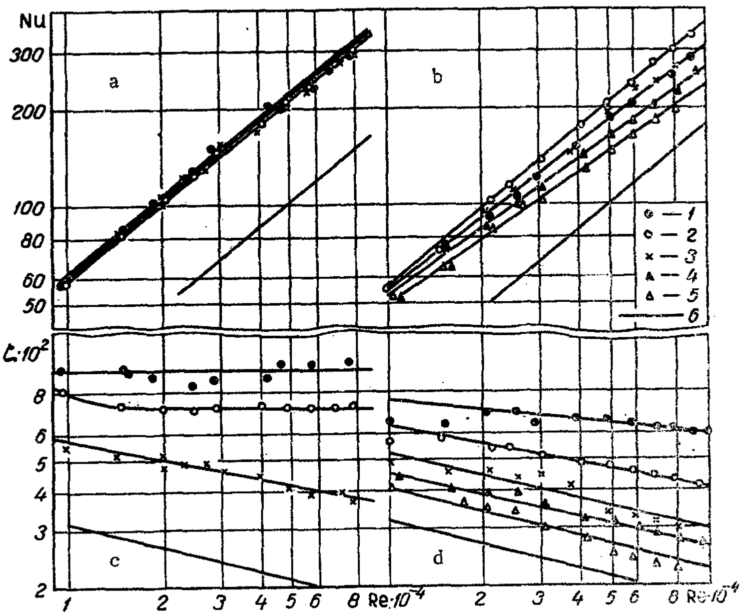

Fig. 2. Curves showing Nu and ζ functions of Re for a = 47.7 mm for channels with sharp leading edges (a, c) and with rounded edges (b, d); 1–6) respectively, channels with a diverging–converging ratio of 5:1, 2:1, 1:1, 1:2, 1:3, and a channel with a constant cross section through the length.

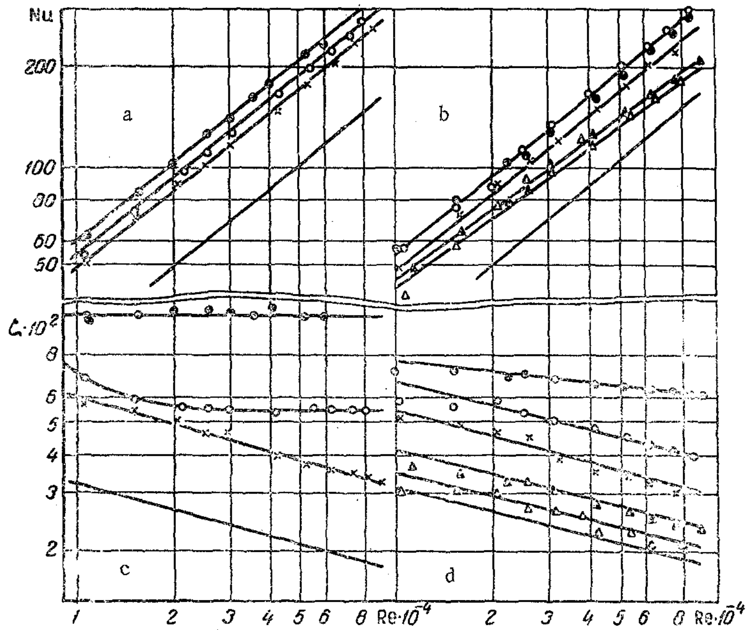

Fig. 3. Curves showing Nu and ζ as functions of Re for a = 33.3 mm for channels with sharp leading edges (a, c) and rounded edges (b, d) (the curves are identified as in Fig. 2).

The average values of Nu were determined separately for the flat plate, the shaped plate, and for the channel as a whole.

In the experiments the value of Re varied within the limits $(10-80) \cdot 10^3$. The value of Pr remained virtually constant (~ 0.7). The arithmetic mean air temperature was taken as the determining temperature. The temperature factor was $T_w/T_f \approx 1.3$.

We can see that the intensity of heat transfer and the hydrodynamic resistance of the channel will be functions of the relative extent of the diverging and converging portions of the channel, as well as of the relationship between the inlet and outlet cross sections of the diffusers, and of the shape of the edge.

To determine the effect of the relationship between the lengths of the diverging and converging portions, we studied the following channels (made up of the diverging and converging sections of diffusers): 5:1 (b = 40 mm, c = 8.0 mm), 2:1 (b = 40 mm, c = 20 mm); 1:1 (b = c = 40 mm), 1:2 (b = 40 mm, c = 80 mm), 1:3 (b = 40 mm, c = 120 mm).

The change in the relationship between the lengths of the diverging and converging portions of the channel indicates a change in the relative effective duration of the positive and negative pressure gradient, as well as a change in the relationship between their absolute values. When the ratio of the diverging section of the diffuser to the converging section of the diffuser is reduced (from 5:1 to 1:3) the positive pressure gradients do not change, and the negative gradients gradually diminish in absolute magnitude (i. e., in intensity of effect) with a simultaneous increase in their effective duration. This makes analysis of the effect of the pressure gradient of different signs more difficult.

To determine the effect of the relationship between the areas of the inlet and outlet cross sections of the diffusers, we varied the distance between the flat and shaped plates. In a uniform diffuser, depending on the relationship between the areas of the inlet and outlet cross sections, various types of flow may arise: flow without separation, flow prior to separation, and separation flow [7]. When a = 47.7 mm in a uniform diffuser with $\gamma = 12°$ we have flow without separation; when a = 33.3 mm we find flow prior to separation; when a = 16.8 mm the flow involves separation, with a stable separation zone at the outlet, its thickness of the order of 4 mm. We can assume that the nature of the flow in the diverging portions of the channel, in basic outline, will correspond to the nature of the flow in the uniform diffuser. In this connection, the flows which arise in a channel made of diverging and converging diffuser sections, for various distances a, will be referred to arbitrarily as free of separation, prior to separation, and with separation, respectively. Thus

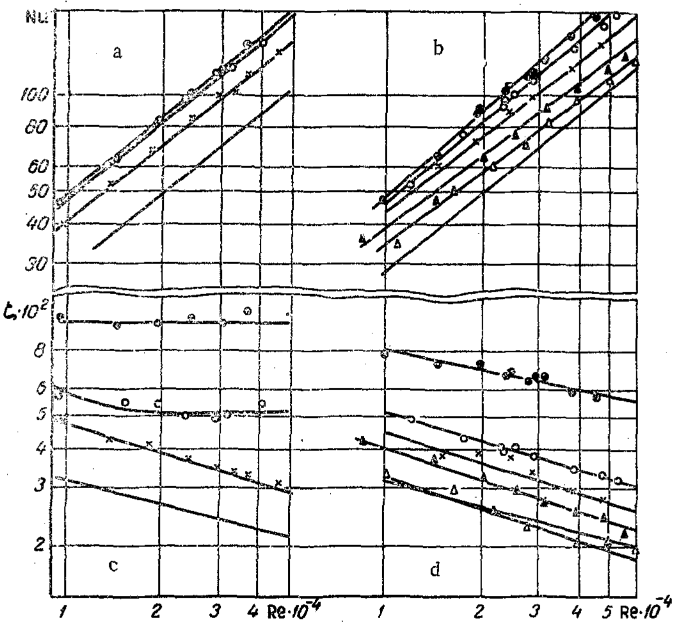

Fig. 4. Curves showing Nu and ζ as functions of Re for a = 16.8 mm for channels with sharp leading edges (a, c) and with rounded edges (b, d) (curves identified as in Fig. 2).

TABLE 1. Values of the Coefficient A_1

a, mm	47,7			33,3			16,8		
Diverging–converging diffusion section	5:1	2:1	1:1	5:1	2:1	1:1	5:1	2:1	1:1
A_1	0,038	0,038	0,038	0,038	0,034	0,031	0,031	0,030	0,026

a change in a is associated with a change in the nature of the flow. Moreover, a reduction in a leads to an increase in the absolute magnitude of the gradients.

To determine the effect of edge shape, we initially investigated channels exhibiting diverging–converging ratios of 5:1, 2:1, 1:1 with sharp leading edges, and then we looked at channels with ratios of 5:1, 2:1, 1:1, 1:2, and 1:3 in which the leading edges were rounded off. We note that with a reduction in the diverging–converging ratio and with an increase in the distance a the effect of the leading edges is reduced.

Let us examine the results from heat-transfer experiments.

For flow of a gas through channels with sharp leading edges (Figs. 2a, 3a, and 4a) we find an approximate proportionality between Nu and $Re^{0.8}$. The intensity of heat transfer is considerably higher (by a factor of approximately 2.1–1.4) than in the case of flow through a rectilinear channel with a constant cross section along the length ($Nu = 0.018\ Re^{0.8}$). For all forms of flow we note a somewhat greater heat-transfer intensity in a channel with a diverging-converging ratio of 5:1. The greatest heat-transfer intensity corresponds to flow without separation.

The experimental heat-transfer data can be approximated by a relationship of the form

$$Nu = A_1\, Re^{0.8}.$$

The values of the coefficient A_1 are given in Table 1.

A slight change in heat-transfer intensity, found with a reduction in the diverging–converging ratio and in the distance a, is apparently explained by the degeneration of the turbulence as a consequence of the negative pressure gradient (they are greater in terms of effective duration and absolute magnitude). Moreover, we can assume that the attachment of the converging diffuser section weakens the turbulent effect of the diffuser as a result of penetration into the latter by an expansion wave.

TABLE 2. Values of the Coefficients A and n

a, mm	47,7					33,3	
Diverging–converging diffuser sections	5:1	2:1	1:1	1:2	1:3	5:1	2:1
A	0,049	0,026	0,049	0,072	0,094	0,039	0,039
n	0,76	0,83	0,76	0,715	0,68	0,79	0,79

a, mm	33,3			16,8				
Diverging–converging diffuser sections	1:1	1:2	1:3	5:1	2:1	1:1	1:2	1:3
A	0,041	0,069	0,067	0,016	0,027	0,040	0,069	0,041
n	0,77	0,705	0,705	0,87	0,81	0,76	0,72	0,735

TABLE 3. Values of the Coefficients B_1 and m_1

a, mm	47,7	33,3	16,8
B_1	0,36	0,65	0,74
m_1	0,20	0,26	0,30

In all cases, the relationship between the average values of Nu for the shaped and flat plates was approximately equal to 1.5. Such a pronounced difference between the heat-transfer intensity for the shaped and flat plate indicates a strong effect on the part of flow nonsymmetry.

For a gas flowing through channels with rounded leading edges the experimental data (Figs. 2b, 3b, and 4b) can be presented in the form of the relationship

$$\mathrm{Nu} = A\,\mathrm{Re}^n.$$

The values of the coefficients A and n are given in Table 2.

In all cases the heat-transfer intensity for a channel with a diverging–converging ratio of 5:1 (with the exception of separation-free flow through a 2:1 channel) is greater than in other channels. With a reduction in the diverging–converging ratio, the relationship between Nu and Re is slightly weakened, which is explained by the gradual degeneration of the turbulence with an increase in the length of the converging portions. The intensity of heat transfer in channels with rounded leading edges is somewhat lower than in channels with sharp leading edges, but substantially greater (by a factor of approximately 2-1.1) than in the case of flow along a rectilinear channel with a cross section that is constant over the length. A change in the diverging–converging ratio and in the distance a, qualitatively speaking, exerts the same influence on the heat-transfer intensity as in the case of channels with sharp leading edges.

The relationship between the average values of Nu for the shaped and flat plates in channels exhibiting a diverging–converging ratio of 5:1, 2:1, 1:1, 1:2, and 1:3 is given, respectively, by 1.7, 1.55, 1.4, 1.25, 1.15. The effect of intensifying transfer on the shaped plate can be explained by the displacement of turbulent vortices toward that plate as a result of the centrifugal forces.

Let us examine the experimental results with respect to hydrodynamic resistance.

With a reduction in the diverging–converging ratio (for a fixed value of a) the static-pressure difference Δp diminishes. In turn, a reduction in the distance, a, for each of the channels, leads to a pronounced increase in Δp.

When a gas flows through a channel with sharp leading edges (Figs. 2c, 3c, and 4c) the function $\xi = \xi(\mathrm{Re})$ is rather complex in form. Thus, in a channel whose diffusing–converging ratio is 5:1 we find self-similarity for all forms of flow (ξ = const). In a channel with a diverging–converging ratio of 2:1 for Re = (10-25) $\cdot 10^3$ we find that ξ is a function of Re, while when $\mathrm{Re} > 25 \cdot 10^{-3}$, ξ = const. Finally, in a channel with a diverging–converging ratio of 1:1 the value of ξ is a rather strong function of Re.

In all cases ξ is substantially larger than in the case of flow through a rectilinear channel with a cross section constant over the length ($\xi = 0.3164\ \mathrm{Re}^{-0.25}$).

An explanation of these unique features of the function $\xi = \xi(\mathrm{Re})$ should be sought in the conditions of flow.

Flow through a diffuser leads to artificial turbulization of the flow, and the level of this turbulization does not correspond to the value of Re [3, 4]. We have not yet finally clarified the mechanism behind this

TABLE 4. Values of the Coefficients B and m

a, mm	47,7					33,3		
Diverging–converging diffuser sections	5:1	2:1	1:1	1:2	1:3	5:1	2:1	1:1
B	0,20	0,37	0,46	0,32	0,27	0,23	0,58	0,62
m	0,105	0,19	0,235	0,22	0,265	0,12	0,24	0,27
a, mm	33,3		16,8					
Diverging–converging diffuser sections	1:2	1:3	5:1	2:1	1:1	1:2	1:3	
B	0,30	0,32	0,49	0,74	0,68	0,78	0,57	
m	0,22	0,24	0,20	0,29	0,30	0,32	0,31	

TABLE 5. Value of the Coefficient k

a, mm	47,7			33,3			16,8		
Diverging–converging diffuser sections	1:1	1:2	1:3	1:1	1:2	1:3	1:1	1:2	1:3
$Re=10\cdot 10^3$	7,00	6,05	5,75	7,70	6,25	5,60	7,00	7,00	6,50
$Re=80\cdot 10^3$	6,86	6,85	6,60	6,90	6,75	6,10	6,35	6,3	5,25

TABLE 6. Results from a Comparison of the Degree of Effectiveness for Channels with Diverging–Converging Sections with Rounded Leading Edges

a, mm	47,7					33,3					16,8				
Diverging–converging diffuser sections	5:1	2:1	1:1	1:2	1:3	5:1	2:1	1:1	1:2	1:3	5:1	2:1	1;1	1:2	1:3
								K_N							
$Re=10\cdot 10^3$	0,27	0,19	0,19	0,18	0,20	0,24	0,20	0,26	0,26	0,26	0,30	0,28	0,30	0,39	0,5
$Re=80\cdot 10^3$	0,47	0,21	0,26	0,36	0,48	0,45	0,23	0,30	0,37	0,46	0,46	0,28	0,37	0,47	0,6
								K_F							
$Re=10\cdot 10^3$	0,53	0,51	0,49	0,50	0,50	0,54	0,51	0,57	0,57	0,55	0,60	0,58	0,60	0,67	0,78
$Re=80\cdot 10^3$	0,72	0,52	0,58	0,66	0,78	0,78	0,54	0,62	0,69	0,76	0,71	0,58	0,69	0,81	0,89
								K_Q							
$Re=10\cdot 10^3$	1,52	1,62	1,60	1,60	1,62	1,55	1,58	1,49	1,50	1,49	1,48	1,46	1,44	1,33	1,20
$Re=80\cdot 10^3$	1,25	1,60	1,52	1,36	1,22	1,29	1,55	1,44	1,34	1,21	1,30	1,46	1,34	1,21	1,12

phenomenon [8]. However, it can be assumed with some validity that the explanation of this effect should be sought in the alternating microseparations and attachments of the boundary layer, which occur in the counterpressure field. There is no doubt that the artificial turbulization of the flow is accompanied by intensified dissipation of energy. In engineering practice, this loss of energy is ocassionally referred to as a loss of expansion pressure.

The pressure losses in the case of flow through a uniform diffuser are generally composed of the losses resulting from friction, expansion, and separation of the flow from the surface. From among these losses, it is only the friction loss that is a function of Re. Given sufficiently high diffuser divergence angles ($\gamma > 6°$), the pressure losses are determined by the losses for expansion, for which we can assume a quadratic law of variation with respect to velocity [9]. As regards the case in which we have flow only through the converging portion of the diffuser, the pressure losses in this case are determined – practically speaking – by the losses resulting from friction.

The existence of sharp leading edges in the case of flow through a channel formed by diverging and converging sections doubtlessly brings the pressure losses closer to the quadratic law. However, as a result of the effect exerted by the flow through the converging section on the flow in the diverging section, this law must be violated, and all the more markedly, the smaller the diverging–converging ratio and the distance a.

In a channel with a diverging–converging ratio of 5:1 we naturally find that diffuser flow predominates. The role of the leading edges is great. The pressure losses are determined by the quadratic law. In a channel with a diverging–converging ratio of 2:1 the role of the flow through the converging portion increases, while the influence of the leading edges is reduced. The quadratic law is found only for $Re > 25 \cdot 10^3$. In a channel with a diverging–converging ratio of 1:1 the flow through the diverging portion ceases to be decisive. The effect of the leading edges is reduced even further and ζ depends strongly on Re.

The experimental data on the drag in a channel with a diverging–converging ratio of 1:1 are approximated by a relationship of the form

$$\zeta = B_1 \mathrm{Re}^{-m_1}.$$

The values of the coefficients B_1 and m_1 are shown in Table 3.

When a gas flows through channels with rounded leading edges (Figs. 2d, 3d, and 4d) the value of ζ in all cases is a function of Re. As the diverging–converging ratio diminishes, ζ becomes an even stronger function of Re. Let us stress that the ζ for a channel with rounded leading edges is considerably smaller than in the case of a channel with sharp leading edges. For most flows ζ is greater than in the case of flow through a rectilinear channel with a cross section that is constant over the length. In the case of preseparation flow and in the case of separation flow through a channel with a diverging–converging ratio of 1:3 (Figs. 3d and 4d) the value of ζ is lower than that calculated, which is a consequence of the reduction in the intensity of momentum transfer in a channel in which flow through the converging portion of the diffuser predominates.

The experimental data on resistance, in this case as well, can be approximated by the relationship

$$\zeta = B \mathrm{Re}^{-m}.$$

The values of the coefficients B and m are given in Table 4.

It should be borne in mind that a reduction in the distance a leads to an increase in the ratio l/d_{eq}: for separation flow we have $l/d_{eq} = 30.2$; for preseparation flow we have $l/d_{eq} = 16$; for separation-free flow we have $l/d_{eq} = 11.6$. Therefore, for channels made up of diverging and converging sections, a reduction in the distance a is associated with a substantial rise in the pressure losses.

It was noted in these experiments that heat transfer greatly influences resistance. Thus, the value of ζ increases by a factor of 1.2-1.35 as the gas is heated, in comparison with the case of isothermal flow, where this effect is intensified as the diverging–converging ratio is reduced.

From the quantitative standpoint, the investigated flows can hardly be reduced to the case of flow through a single diffuser, and even less can they be presented as the result of a simple addition of the properties of flows through a single diverging section and a single converging section of a diffuser.

When we are dealing with the diverging section alone, the behavior of the flow as regards separation (and the corresponding pressure loss) depends on the fullness of the velocity profile [10, 11] and on the degree of turbulence at the channel inlet [12]. Here, the more complete the velocity profile, the smaller the pressure losses.

At first glance, it seems that greater profile fullness can be achieved by increasing the length of the converging portions. However, this leads to a reduction in flow turbulence and it is impossible directly to predict which of the two trends will predominate and, consequently, it is unclear what consequences will actually result, in this sense, from an increase in the length of the flow through the converging section.

The drift of the flow-separation points in the diffuser, as a function of Re [7], leaves its mark on the flow pattern in a field of a longitudinal pressure gradient of variable sign.

Particular interest is shown in the problem of the applicability of the Reynolds analogy to the flow in the field of a longitudinal pressure gradient of variable sign.

The experiment shows that the Reynolds analogy is retained approximately for all forms of flow through a channel with a diverging–converging ratio of 1:1, where the leading edges are sharp, and for flow through a channel with a ratio of 2:1, where the leading edges are rounded.

For all other cases, the analogy breaks down. Thus, for all forms of flow through a channel with a diverging–converging ratio of 5:1, where the leading edges are both sharp and rounded, and for flow through a channel with a ratio of 2:1, with sharp leading edges, the analogy breaks down with respect to the momentum transfer. Conversely, for all forms of flow through channels with ratios of 1:1, 1:2, and 1:3, with rounded leading edges, we find pronounced breakdown of the analogy in favor of the transfer of heat, and this effect is intensified as the diverging–converging ratio is reduced. Table 5 shows the values of the proportionality factor k for this case in the relationship

$$\mathrm{St} = \frac{\zeta}{k}.$$

Let us recall that for the gradient-free flow of an ideal gas (Pr = 1) we have k = 8. As we can see, the greatest breakdown of the analogy occurs in a channel with a diverging–converging ratio of 1:3.

The breakdown of the analogy in favor of momentum transfer, which is found in channels with sharp leading edges and predominant diffuser flow, is explained by the increase in the relative role of those forms of energy dissipation which are not associated with the transfer of momentum between the flow and the surface.

The breakdown of the analogy in favor of heat exchange transfer in channels with rounded edges and in which flow through the converging section of the diffuser predominates is explained, apparently, by the specific features of the hydrodynamic situation which arises in the case of flow through such channels. The turbulent vortices which are generated in the diverging portions of the diffuser enter the converging sections as a consequence of the negative pressure gradients (small in absolute magnitude, but rather large in terms of effective duration). Here, the relationship between the forces accelerating the flow and the frictional forces, apparently, is such that the turbulent vortices, as they degenerate, behave as vortices of free turbulence, and thus the effect of the walls is weakened.

When we compare the experimental data on heat transfer and resistance for channels made up of diverging and converging sections with the theoretical data for a channel with a cross section that is constant over the length – this comparison carried out in accordance with [13] – we find that the channels investigated here are highly effective.

In the case of channels made up of the diverging and converging section the power consumption needed to move the heat carrier (for a fixed heat flow and surface), and the surface (for a fixed flow and a fixed consumption of power), are substantially smaller than in the case of channels with a lateral cross section that is constant over the length, while the heat flow (in the case of fixed power consumption and surface), conversely, is greater. Table 6 shows the results from a comparison undertaken for channels made up of diverging–converging sections with rounded edges, and these proved to be substantially more effective than channels with sharp leading edges.

It can thus be stated that the investigated channels made up of diverging and converging diffuser sections are more effective and can be used successfully in the design of heat-exchange equipment.

NOTATION

Re	is the Reynolds number;
γ	is the divergence angle for the diverging portion of the diffuser;
h	is the plate height;
l	is the plate length;
b	is the length of the diverging portion of the diffuser;
c	is the length of the converging portion of the diffuser;
a	is the distance between the flat and shaped plates;
ζ	is the coefficient of hydrodynamic resistance;
Nu	is the Nusselt number;
w	is the average air flow rate in the inlet section of the diffuser;
d_{eq}	is the equivalent diameter of the inlet section of the diffuser;
ν	is the kinematic coefficient of viscosity for the air;
Δp	is the static pressure difference across the channel length;
ρ	is the air density;
α	is the heat-transfer coefficient (average value) for the wall and the air;
λ	is the thermal conductivity of the air;

Pr	is the Prandtl number;
T_w and T_f	respectively, are the thermodynamic temperatures of the surface and the air;
St	is the Stanton number;
K_N, K_F, and K_Q	are, respectively, the power-consumption ratio (for a fixed heat flow and surface), the surface ratio (for a fixed heat flow and power consumption), and the heat-flow ratio (for fixed power consumption and surfaces) for a channel made up of diverging and converging diffuser sections and a channel with a cross section that is constant over the length.

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